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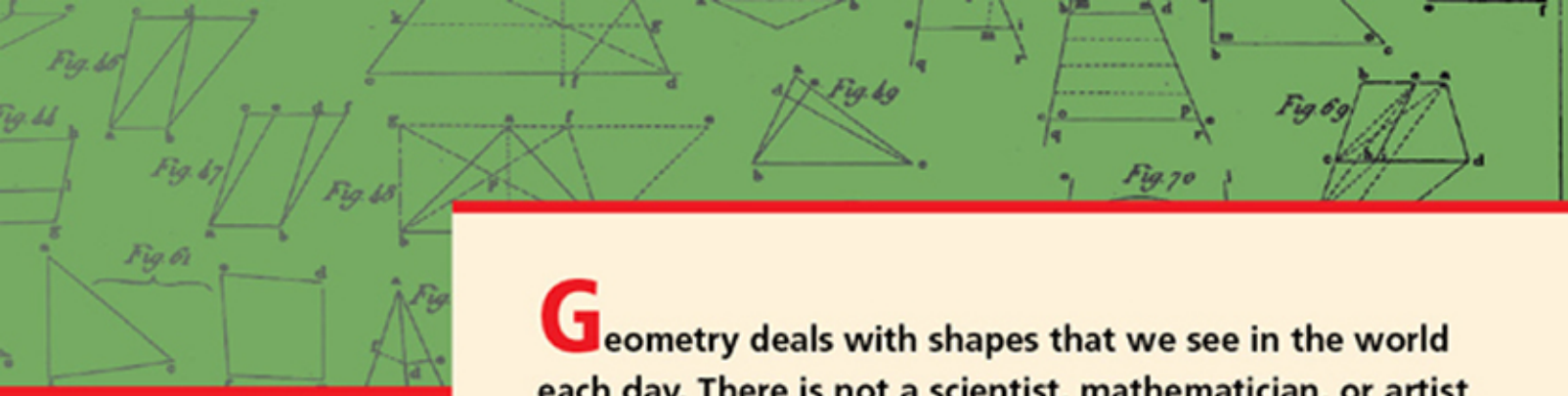

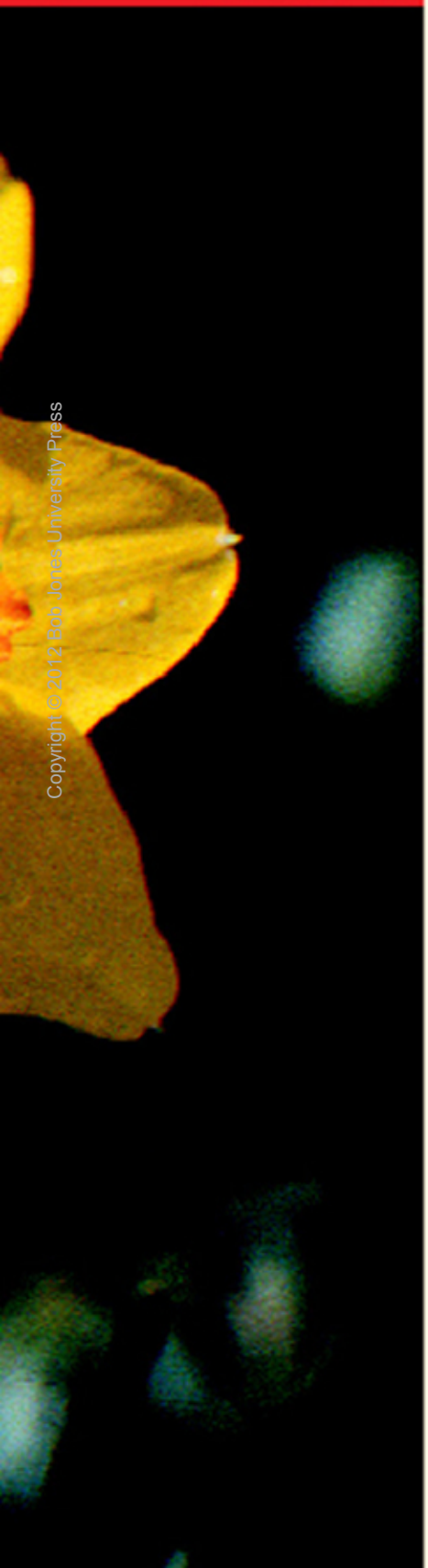

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Geometry deals with shapes that we see in the world each day. There is not a scientist, mathematician, or artist in the world who can create the loveliness displayed in the daffodil pictured on this page.

The flower of the daffodil displays a hexagon pattern. For you see, the greatest master of geometry is our God.

Do you think you are sitting in geometry class today because of chance? You're not. Through the discipline of geometry, God has something to teach you about yourself and, more importantly, about Him. As you glimpse God's order through geometry, you should marvel at His wisdom. And you should apply yourself with all diligence to "do all to the glory of God," which includes geometry class—today.

After this chapter you should be able to

1. express sets, subsets, and elements of sets with proper symbols.
2. perform set operations: unions, intersections, complements.
3. define terms and list the three undefined terms.
4. discuss points, lines, and their relationships, using proper symbols.
5. distinguish between postulates and theorems.
6. explain the criteria for an ideal geometric system.
7. state incidence postulates.
8. prove incidence theorems.

1.3 Undefined Terms and Definitions

The peacock fans his tail to form a plane containing points, lines, and circles.

Where do you start a mathematical system such as geometry? To begin the study of geometry, you must understand what basic geometrical words mean. A good definition of some words is hard to find. What is a good definition anyway? A definition is a statement of meaning. A good definition must have the following characteristics to successfully convey the meaning of the term.

1. **Clear**. The definition must communicate the point and state the term being defined. Avoid vague or ambiguous language.
2. **Useful**. The definition must use only words that have been previously defined or are commonly accepted as undefined.
3. **Precise**. The definition must be accurate and reversible. Identify the class to which the object belongs and its distinguishing characteristics.
4. **Concise**. The definition must be a good sentence and use good grammar. Stick to the point and avoid unnecessary words.
5. **Objective**. The definition must be neutral. Avoid emotional words, figures of speech, and limitations of time or place.

EXAMPLE Evaluate this definition. *Space* is the set of all points.

Answer To determine if this is a good definition of space, we need to analyze each of the five characteristics.

1. **Clear**. The object being defined is named in the definition. It is not ambiguous.
2. **Useful**. Look at all the words in the definition. Have they all been defined? At this point in our study, we must say no. The word *set* was discussed in the first two sections of the chapter, but has the word *point* been defined?
3. **Precise**. The definition describes space accurately. Space is classified as a set and is described as the particular set containing all points. The definition is also reversible. We could say, "The set of all points is space."

Continued

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4. **Concise**. This definition contains only necessary words and uses proper grammar.

5. **Objective**. This definition uses appropriate objective language.

We can conclude from this analysis that this definition of *space* would be a good one if the word *point* were defined.

Consider the word *point*. Can it be defined? Most of you certainly have some idea of what a *point* is. If we define *point*, some words in that definition would also need to be defined, and this process could continue forever. To avoid this needless waste of time and energy, mathematicians have agreed to accept some undefined terms as basic building blocks for a mathematical system. Of course, they desire that there be as few undefined terms as possible. Since the term *point* is one of these basic terms, the definition of space is a good definition.

In geometry there are three undefined terms that can be described but not defined: *point*, *line*, and *plane*.

Undefined term	Description	Illustration	Notation
Point	Spot; an object with no dimensions, length, width, or thickness; a location in space	pinpoint	denoted by a capital letter; location marked by a dot
Line	Straight; an object that extends infinitely in one dimension; has length but no width or thickness	laser beam	$\overleftrightarrow{CD}$; or a lowercase script letter 
Plane	Flat; an object that extends infinitely in two dimensions; has length and width but no thickness	thin sheet of glass	plane *k*; denoted by a lowercase letter 

These descriptions are not definitions. The descriptions should simply help you visualize concepts. In reality, points, lines, and planes cannot actually be seen.

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The sheer cliff face, down which this man is rapelling, can be represented by a plane.

Likewise, there is a starting place for Christian doctrine. There are certain things that the believer must accept as truth although he may not fully understand them. For example, the believer must believe that the Bible is the verbally inspired, inerrant Word of God. He must believe in and accept the atoning death and shed blood of Jesus Christ for salvation. Although he may not thoroughly understand these things, his faith in these basic truths forms the foundation on which he can build a strong spiritual life.

▶ A. Exercises

Check all the characteristics of a good definition and determine which of the following definitions are good. If a definition is not good, explain why.

1. A frog is an amphibian.
2. A noun is a word used to name a person, place, thing, quality, or action.
3. A glove is a covering for the hand.
4. Plasma is the clear, yellowish liquid part of the blood.
5. Nine is the sum of six and three.
6. An empty set is any set.
7. An atheist is like when some crazy nut denies that God exists.
8. Love is unselfish concern for the best interests of another.
9. A retable is a reredos.
10. Braille is a system of writing and printing for blind people that consists of different patterns of raised dots that represent letters, words, numbers, and punctuation marks.

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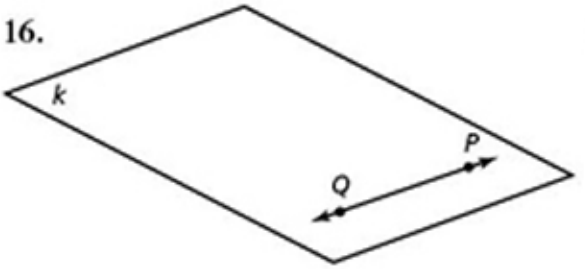
Illustrate each idea.

11. point K
12. plane p
13. line l
14. $\overleftrightarrow{AC}$

Write a sentence describing each illustration.

15.

16.



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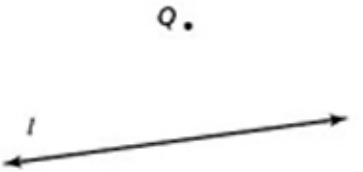

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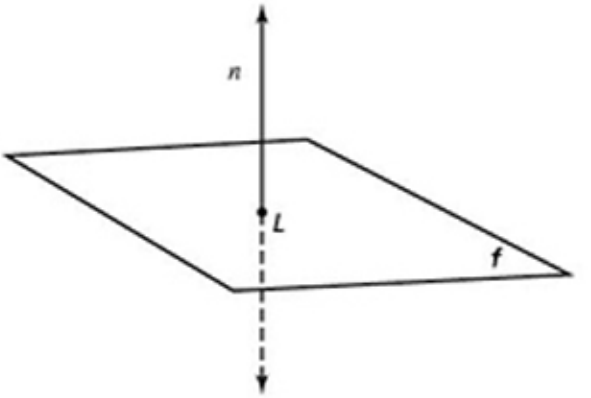

▶ B. Exercises

Define the following.

19. king
20. pen
21. mosaic
22. emu

▶ C. Exercises

23. Evaluate the definition of *element* of a set from page 2.
24. Correct any definitions in exercises 1-10 that were not good.

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▶ Dominion Thru Math

Study the daffodils on the opening pages of this chapter and answer the questions.

25. How many petals are there, and what figure do their tips form?
26. What figure is formed by endpoints of every other petal?
27. What shape do you see near the center with the orange edges?
28. Even though the petals are formed by slightly curved pieces, what general shape are they?
29. What shapes are the little bundles of pollen pistils in the center?

Study the flowers and the butterfly on page 498 and answer the questions.

30. How many petals are there, and what figure do the tips form?
31. What is the basic shape of each petal?
32. What shape is in the white center of the flower?
33. What is different about these pollen pistils as compared to those of the daffodil?

Cumulative Review

State in words the most specific relationship that you can between each pair of sets. Express as many of the relations in symbols as you can. Consider the integers as the universal set.

34. $A = \{1, 3, 5\}$
    $B = \{x \mid x \text{ is an odd integer}\}$
35. $A = \{x \mid x \text{ is a prime number greater than } 2\}$
    $B = \{x \mid x \text{ is an even integer}\}$
36. $A = \{x \mid x \text{ is the square of an integer}\}$
    $B = \{0, 1, 4, 9, 16, 25, \ldots\}$
37. $A = \{1, 11, 21, 31\}$
    $B = \{-11, 1, 11, 121, 1331\}$
38. $A = \{x \mid x > 2\}$
    $B = \{x \mid x \leq 2\}$

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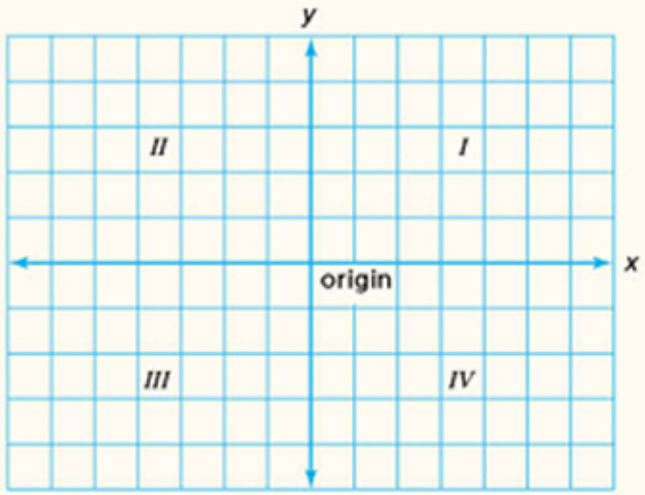
1 Analytic Geometry

Graphing Points

You may be asking yourself, "How does any of this geometry relate to what I learned in algebra?" *Analytic geometry* makes a connection between the figures that you see in geometry and the equations that you saw in algebra. In each chapter of this book, you will find a feature on analytic geometry to help you see the connection.

Points are graphed on a *Cartesian plane,* named for René Descartes, who was the first mathematician to connect geometric figures and equations. Every point in a plane is identified by an *ordered pair,* such as $(3, -2)$, which is measured from a reference point called the *origin*. The origin is the intersection of two perpendicular lines called the *axes*. The horizontal line is called the x-axis, and the vertical line is called the y-axis. The axes divide the Cartesian plane into four *quadrants*.



The origin is the ordered pair $(0, 0)$. You can graph any other ordered pair by counting from the origin. The first number in an ordered pair, such as the 3 in $(3, -2)$, represents the x-value, or distance of the particular point in the horizontal direction. The second number is the y-value, or vertical distance.

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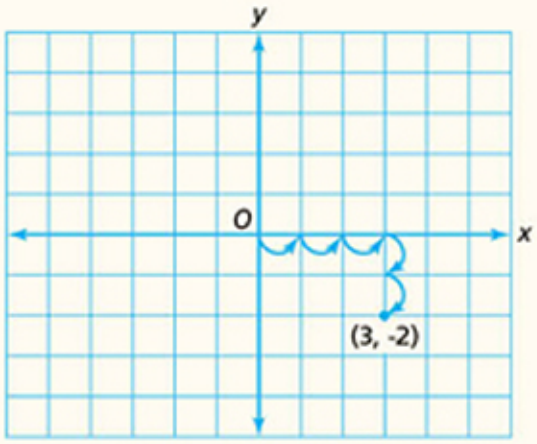
To locate the point (3, −2), start at the origin and move 3 units right and 2 units down. Be sure to label the point.

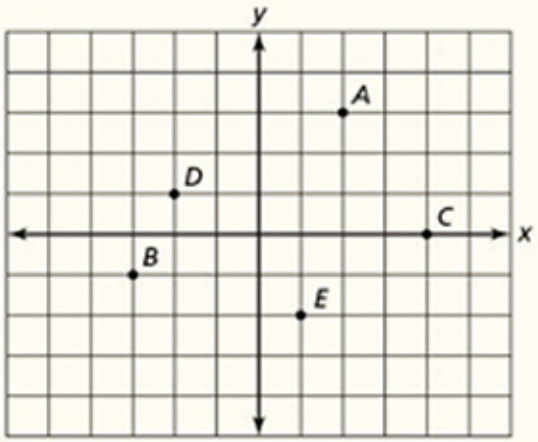


Each point of the plane has an *x*-value and a *y*-value. By relating geometric points to these algebraic variables, you can derive geometric truths. In this way, analytic geometry uses algebra to study geometry.

▶ Exercises

Give the coordinates of each point.

1. *A*
2. *B*
3. *C*
4. *D*
5. *E*



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