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LESSON 1 The Real Numbers • Fundamental Concept Review

1.A the real numbers

The numbers that we naturally use to count make up the set called the **natural numbers,** the **counting numbers,** or the **positive integers.** We use the symbol $\mathbb{N}$ to represent this set.

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

When we include the number 0 with these numbers, we get the whole numbers.

$$\text{Whole Numbers} = \{0, 1, 2, \ldots\}$$

The negatives of the natural numbers are called the **negative integers.** The number zero together with the positive and the negative integers forms the set of **integers.** The symbol $\mathbb{Z}$ is often used to represent this set. (This symbol comes from the first letter in the German word *Zahlen,* which means "integers.")

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Any number that can be expressed as a quotient (fraction) using two integers is called a **rational number.** (Remember that zero cannot be a divisor.) We use the symbol $\mathbb{Q}$ (for quotient) to designate this set. The following numbers are rational numbers:

$$0, 4, 0.0021, -\frac{7}{23}, \frac{45}{14}, -4.16\overline{32}, 1.12121212\ldots$$

Any decimal number that cannot be written as a quotient of integers is called an **irrational number.** We do not have a symbol for the set of irrational numbers. Examples of irrational numbers are:

$$\sqrt{2}, \pi, e, \sqrt[3]{13}, \sqrt[5]{41}$$

The set of **real numbers** includes all members of the set of rational numbers and all members of the set of irrational numbers. We use the symbol $\mathbb{R}$ to represent the set of real numbers. Using the symbol $\subset$ to mean "is a subset of," we can write

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Every natural number is an integer. Every integer is a rational number. And every rational number is a real number.

In the chart below, we summarize what we have learned.

REAL NUMBERS

IRRATIONAL NUMBERS

Real numbers that cannot be expressed as a fraction using two integers, such as: $\sqrt[3]{5}, \sqrt{2}, -\sqrt{2}, 2\sqrt{2}, \pi, 3\pi$, etc.

RATIONAL NUMBERS

Real numbers that can be expressed as a fraction using two integers, such as: $\frac{1}{2}$, 0.3, $-\frac{2}{3}$, etc.

INTEGERS

Set of whole numbers and the opposites of the natural numbers:

$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$

WHOLE NUMBERS

Set of natural numbers and the number 0

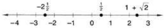
$0, 1, 2, 3, 4, \ldots$

NATURAL OR COUNTING NUMBERS

$1, 2, 3, 4, \ldots$

Students are advised against using this chart to draw conclusions about how large a set is relative to other sets. Rather, use it as a reference to remember what kinds of numbers are in each set and to get information about which sets are subsets of others.

The real numbers are an ordered set, for the members of the set of real numbers can be arranged in order, which we indicate when we draw a real number line.



Each point on the number line is associated with a unique number called the **coordinate** of the point. When we graph a number, we place a dot on the number line to indicate the position of the point that has this number as its coordinate. On the number line above, the numbers $\frac{1}{2}$, $1 + \sqrt{2}$, and $-2\frac{1}{2}$ are graphed. Below we list the order properties of the real numbers.

ORDER PROPERTIES OF THE REAL NUMBERS

Let x, y, and z represent real numbers.

1. **Trichotomy.** Exactly one of the following is true:
   $x < y$ or $x = y$ or $x > y$
2. **Transitivity.** If $x < y$ and $y < z$, then $x < z$.
3. **Addition.** If $x < y$, then $x + z < y + z$.
4. **Multiplication.** If z is positive and $x < y$, then $xz < yz$. If z is negative and $x < y$, then $xz > yz$.
5. **Reciprocal.** If x and y are positive and $x < y$, then $\frac{1}{x} > \frac{1}{y}$.

The real numbers also constitute a **field.** The properties of fields are shown below. Note that the symbol $\in$ means "is an element of."

FIELD PROPERTIES

Let x, y, and z be elements of a field F.

1. **Closure laws.** $x + y \in F$ and $xy \in F$
2. **Commutative laws.** $x + y = y + x$ and $xy = yx$
3. **Associative laws.**
   $x + (y + z) = (x + y) + z$ and $x(yz) = (xy)z$
4. **Distributive law.** $x(y + z) = xy + xz$
5. **Identity elements.** There are two distinct numbers 0 and 1 satisfying $x + 0 = x$ and $x \cdot 1 = x$.
6. **Inverses.** Each number x has an additive inverse (also called a negative), $-x$, satisfying $x + (-x) = 0$. Also, each number x except 0 has a multiplicative inverse (also called a reciprocal), x^{-1}, satisfying $x \cdot x^{-1} = 1$.

1.B fundamental concept review

To be a successful calculus student, you must review some fundamental concepts from algebra. Rather than present an expository review, we work problems whose solutions require us to apply the concepts.

example 1.1 Solve $y = v\left(\frac{a}{x} + \frac{b}{mc}\right)$ for c.

solution The solution can be found using six steps: (1) eliminate parentheses, (2) multiply by the least common multiple of the denominators, (3) simplify, (4) put all terms containing c on one side of the equals sign, (5) factor c, and (6) divide. In each step we assume that no denominator equals zero.

(1) $y = \frac{va}{x} + \frac{vb}{mc}$ eliminated parentheses

(2) $xmc \cdot y = xmc \cdot \frac{va}{x} + xmc \cdot \frac{vb}{mc}$ multiplied by LCM of denominators

(3) $xmcy = mcva + xvb$ simplified

(4) $xmcy - mcva = xvb$ rearranged

(5) $c(xmy - mva) = xvb$ factored

(6) $\mathbf{c = \frac{xvb}{xmy - mva}}$ divided

example 1.2 Simplify: (a) $\frac{x}{a + \frac{m}{1 + \frac{c}{d}}}$ (b) $\frac{\frac{a}{x^2} + \frac{b}{x}}{\frac{m}{x^2} + \frac{k}{xc}}$

solution (a) When there is no equals sign, the denominators cannot be eliminated. However, this expression can be written as a simple fraction using the following four steps: (1) add, (2) simplify, (3) add, and (4) simplify.

(1) $\frac{x}{a + \frac{m}{\frac{d + c}{d}}}$ added

(2) $\frac{x}{a + \frac{md}{d + c}}$ simplified

(3) $\frac{x}{\frac{a(d + c) + md}{d + c}}$ added

(4) $\mathbf{\frac{x(d + c)}{a(d + c) + md}}$ simplified

(b) There is no equals sign in this expression, so the denominators cannot be eliminated. We (1) add in the numerator and denominator and (2) simplify.

(1) $\frac{\frac{a + bx}{x^2}}{\frac{mc + kx}{x^2 c}}$ added

(2) $\mathbf{\frac{c(a + bx)}{mc + kx}}$ simplified

example 1.3 Simplify: $\dfrac{4 + \sqrt{2}}{3 - 2\sqrt{2}}$

solution We multiply above and below by $3 + 2\sqrt{2}$, which is the conjugate of the denominator, and then simplify.

$$\frac{4 + \sqrt{2}}{3 - 2\sqrt{2}} \cdot \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} = \frac{16 + 11\sqrt{2}}{9 - 8} = \mathbf{16 + 11\sqrt{2}}$$

example 1.4 Simplify: $3\sqrt{\dfrac{3}{2}} - 4\sqrt{\dfrac{2}{3}} - \sqrt{24}$

solution First we rationalize the denominators.

$$3 \cdot \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - 4 \cdot \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} - 2\sqrt{6} = \frac{3\sqrt{6}}{2} - \frac{4\sqrt{6}}{3} - 2\sqrt{6}$$

We finish by adding these three terms, using 6 as a common denominator.

$$\frac{9\sqrt{6}}{6} - \frac{8\sqrt{6}}{6} - \frac{12\sqrt{6}}{6} = -\frac{\mathbf{11\sqrt{6}}}{\mathbf{6}}$$

example 1.5 Simplify: (a) $\dfrac{y^{x+3}y^{x/2-1}z^{a}}{y^{(x-a)/2}z^{(x-a)/3}}$ (b) $x^{3/4}\sqrt{xy}\,x^{1/2}\sqrt[3]{x^4}$

solution (a) First we collect powers of like bases. Then we add the exponents.

$$y^{x+3+x/2-1-x/2+a/2}z^{a-x/3+a/3} = \mathbf{y^{x+2+a/2}z^{4a/3-x/3}}$$

(b) We replace the radicals with fractional exponents. Then we add the exponents of like bases.

$$x^{3/4}x^{1/2}y^{1/2}x^{1/2}x^{4/3} = \mathbf{x^{37/12}y^{1/2}}$$

example 1.6 Factor: $4a^{3m+2} - 16a^{3m}$

solution If each term is written in factored form, the common factor $4a^{3m}$ can be determined by inspection. We extract the common factor and finish by factoring $a^2 - 4$.

$$(4a^{3m})a^2 - (4)(4a^{3m}) = 4a^{3m}(a^2 - 4) \qquad \text{common factor}$$
$$= \mathbf{4a^{3m}(a + 2)(a - 2)} \qquad \text{factored } a^2 - 4$$

example 1.7 Factor: (a) $8a^3 - b^3c^6$ (b) $m^3 + x^3y^6$

solution (a) The difference of two cubes $F^3 - S^3$ can be factored as $(F - S)(F^2 + FS + S^2)$.

$$8a^3 - b^3c^6 = (2a)^3 - (bc^2)^3$$
$$= \mathbf{(2a - bc^2)(4a^2 + 2abc^2 + b^2c^4)}$$

(b) The sum of two cubes $F^3 + S^3$ has similar factorization:

$$F^3 + S^3 = (F + S)(F^2 - FS + S^2)$$

Therefore:

$$m^3 + x^3y^6 = (m)^3 + (xy^2)^3$$
$$= \mathbf{(m + xy^2)(m^2 - mxy^2 + x^2y^4)}$$

example 1.8 Simplify: (a) $\dfrac{14!}{6!\,11!}$ (b) $\dfrac{N!}{(N-2)!}$ (c) $\displaystyle\sum_{j=0}^{3} \frac{2^j}{j+1}$ (d) $\displaystyle\sum_{i=1}^{4} 3$

solution Recall that $N!$, read "N factorial," is defined to be the product of the integers from 1 to N.

$$N! = N \cdot (N - 1) \cdot (N - 2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$$

(a) $\dfrac{14!}{6!\,11!} = \dfrac{\overset{7}{\cancel{14}} \cdot 13 \cdot \cancel{12} \cdot \cancel{11!}}{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{11!}} = \dfrac{7 \cdot 13}{30} = \mathbf{\dfrac{91}{30}}$

(b) $\dfrac{N!}{(N-2)!} = \dfrac{N \cdot (N-1) \cdot (N-2)!}{(N-2)!} = N \cdot (N-1) = N^2 - N$

The symbol Σ indicates summation.

(c) $\displaystyle\sum_{j=0}^{3} \frac{2^j}{j+1} = \frac{2^{(0)}}{(0)+1} + \frac{2^{(1)}}{(1)+1} + \frac{2^{(2)}}{(2)+1} + \frac{2^{(3)}}{(3)+1} = 1 + 1 + \frac{4}{3} + 2 = \mathbf{\frac{16}{3}}$

(d) $\displaystyle\sum_{i=1}^{4} 3 = 3 + 3 + 3 + 3 = \mathbf{12}$

example 1.9 Compare (assume $a \neq 0$): A. $\dfrac{1}{a}$ B. a^{-1}

In comparison problems throughout this text, the answer is A if quantity A is greater, B if quantity B is greater, C if quantities A and B are equal, and D if insufficient information is provided to determine which quantity is greater.

solution Quantities A and B are equal since a^{-1} is another way of writing $\dfrac{1}{a}$. Therefore, the answer is **C.**

problem set 1

In Saxon textbooks it is customary to give problems that cover only those concepts discussed in the text itself. However, in the early problem sets we will not follow this custom. For example, in Problem Set 1, problems 1–4, 13, 24, and 25 are not discussed in the lesson. Students who have difficulty with any of the review problems in these early lessons should refer to earlier texts in the Saxon series.

For problems 1–4, the answer is A if quantity A is greater, B if quantity B is greater, C if quantities A and B are equal, and D if insufficient information is provided to determine which quantity is greater.

1. Compare: A. $7\frac{1}{4}$ ft^2 B. 0.8 yd^2

2. Given that $x = t$, compare: A. $7(2t - 2x)$ B. $-6(3t - 3x)$

3. Given that $4 < x < 9$ and $2 < y < 14$, compare: A. x B. y

4. Given that a is the average of 3 and 6, compare: A. $3a$ B. $a + 6$

5. Solve for R_1: $\dfrac{m}{x} = y\left(\dfrac{1}{R_1} + \dfrac{a}{R_2}\right)$

Simplify the expressions in problems 6–13.

6. $a + \dfrac{1}{a + \dfrac{1}{a}}$

7. $\dfrac{1}{a + \dfrac{1}{x + \dfrac{1}{m}}}$

8. $\dfrac{x^2 y}{1 + m^2} + \dfrac{x}{y}$

9. $\dfrac{4 - 3\sqrt{2}}{8 - \sqrt{2}}$

10. $\dfrac{x^a y^{a+b}}{x^{-a/2} y^{b-1}}$

11. $\dfrac{m^{x+2} b^{x-2}}{m^{2x/3} b^{-3x/2}}$

12. $\sqrt{xy}\,x^{2/3} y^{-3/2}$

13. Solve: $\begin{cases} 2x + 3y = -4 \\ x - 2z = -3 \\ 2y - z = -6 \end{cases}$

Factor the expressions in problems 14–19.

14. $a^2x - a^2 - 4b^2x + 4b^2$ **15.** $16a^{4m+3} - 8a^{2m+3}$ **16.** $a^2b^{2x+2} - ab^{2x+1}$

17. $9x^2 - y^4$ **18.** $a^6 - 27b^3c^3$ **19.** $x^3y^6 + 8m^{12}$

Simplify the expressions in problems 20–23.

20. $\dfrac{12!}{8!\,4!}$ **21.** $\dfrac{n(n!)}{(n+1)!}$ **22.** $\sum_{i=1}^{3} 4$ **23.** $\sum_{m=0}^{3} \dfrac{3^m}{m+1}$

24. Find the surface area of a sphere whose volume is $\frac{4}{3}\pi$ cubic meters.

25. Find the volume of a right circular cone whose base has an area of 4π square centimeters and whose height is 4 centimeters.

LESSON 2 More Concept Review • The Graphing Calculator

2.A more concept review

We continue reviewing fundamental concepts.

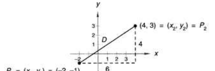
example 2.1 Find the coordinates of the point halfway between (–4, 7) and (13, 5).

solution The x-coordinate of the midpoint is the average of the x-coordinates, and the y-coordinate of the midpoint is the average of the y-coordinates.

$$x_m = \frac{-4 + 13}{2} = \mathbf{\frac{9}{2}} \qquad y_m = \frac{7 + 5}{2} = \mathbf{6}$$

example 2.2 Find the distance between (4, 3) and (–2, –1).

solution First we graph the points. The distance between the points is found by using the distance formula, which is an extension of the Pythagorean theorem. We arbitrarily choose point (–2, –1) to be P_1 and (4, 3) to be P_2.



The distance between P_1 and P_2 is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-2-4)^2 + (-1-3)^2}$$
$$= \sqrt{(-6)^2 + (-4)^2}$$
$$= \sqrt{52} = \mathbf{2\sqrt{13}}$$

example 2.3 Use the point-slope form of the equation of a line to write the slope-intercept form of the equation of the line that passes through (–2, 4) and has a slope of $-\frac{2}{3}$.

TEST 1

1. $(\cos^2\theta)(\sec\theta)(\tan^2\theta)(\csc\theta)$
(4)

$= (\cos^2\theta)\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= \frac{\sin\theta}{\cos\theta} = \mathbf{\tan\theta}$

The correct choice is **C.**

2. $m^6x^3 + n^3y^9$
(1)

$= (m^2x)^3 + (ny^3)^3$

$= \mathbf{(m^2x + ny^3)(m^4x^2 - m^2xny^3 + n^2y^6)}$

The correct choice is **B.**

3. $y - 2 = 7[x - (-3)]$
(2)

$y - 2 = 7(x + 3)$

$y - 2 = 7x + 21$

$\mathbf{y = 7x + 23}$

The correct choice is **D.**

4. $\cos^2\frac{\pi}{3} + \tan\frac{\pi}{4} = \left(\frac{1}{2}\right)^2 + 1 = \frac{1}{4} + 1 = \mathbf{\frac{5}{4}}$
(4)

5. $\sum_{n=0}^{5} n^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2$
(1)

$= 0 + 1 + 4 + 9 + 16 + 25$

$= \mathbf{55}$

6. $\begin{cases} x^2 + y^2 = 9 \\ x - y = 1 \end{cases}$
(2)

Solve the second equation for y: $y = x - 1$

Substitute into the first equation.

$$x^2 + (x-1)^2 = 9$$
$$x^2 + x^2 - 2x + 1 = 9$$
$$2x^2 - 2x - 8 = 0$$
$$x^2 - x - 4 = 0$$
$$x = \frac{1 \pm \sqrt{17}}{2}$$

$x = \frac{1}{2} + \frac{\sqrt{17}}{2}$ or $x = \frac{1}{2} - \frac{\sqrt{17}}{2}$

$y = -\frac{1}{2} + \frac{\sqrt{17}}{2}$ $\quad y = -\frac{1}{2} - \frac{\sqrt{17}}{2}$

$\mathbf{\left(\frac{1}{2} + \frac{\sqrt{17}}{2}, -\frac{1}{2} + \frac{\sqrt{17}}{2}\right)}$ and

$\mathbf{\left(\frac{1}{2} - \frac{\sqrt{17}}{2}, -\frac{1}{2} - \frac{\sqrt{17}}{2}\right)}$

7. $\frac{50!}{46!\,4!} = \frac{50(49)(48)(47)(46!)}{46!\,(4!)} = \frac{50(49)(48)(47)}{4(3)(2)(1)}$
(1)

$= 50(49)(2)(47) = 100(49)(47) = 100(2303)$

$= \mathbf{230{,}300}$

8. Contrapositive: **If $a + b \neq 5$, then $ab \neq 4$.**
(3)

Converse: **If $a + b = 5$, then $ab = 4$.**

Inverse: **If $ab \neq 4$, then $a + b \neq 5$.**

9. $\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}} = \frac{1}{1 + \frac{1}{\frac{3}{2}}} = \frac{1}{1 + \frac{2}{3}} = \frac{1}{\frac{5}{3}} = \mathbf{\frac{3}{5}}$
(1)

10. $3y = 2x - 3$
(2)

$y = \frac{2}{3}x - 1$

The slope of a line perpendicular to this one is $-\frac{3}{2}$.

$y - 3 = -\frac{3}{2}(x + 1)$

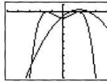
$y - 3 = -\frac{3}{2}x - \frac{3}{2}$

$\mathbf{y = -\frac{3}{2}x + \frac{3}{2}}$

11. The roots of $ax^2 + bx + c = 0$ are
(2)

$x = \mathbf{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$.

12. (a)
(2)



There are two intersection points, **(−0.146, −3.916)** and **(−2.828, −35.960).**

(b) The distance between the two points is:

$\sqrt{(-0.146 + 2.828)^2 + (-3.916 + 35.960)^2}$

$\approx \mathbf{32.156}$

20. $x^3y^6(1 - 2xy^2)(1 + 2xy^2 + 4x^2y^4)$

21. n

22. 8

23. $-\frac{16}{3}$

24. $5\sqrt{3}\text{ m}^3 \approx 8.6603\text{ m}^3$

25. D

Problem Set 3

1. If the switch is on, then the light is on.

2. If the light is not on, then the switch is not on.

3. If the switch is not on, then the light is not on.

4. If x is not a complex number, then x is not a real number.

5. $y = -2x + 6$

6. $(x + 3)^2 + 4 = 0$

7. $\frac{3}{2} \pm \frac{\sqrt{37}}{2}$

8. $(2 + \sqrt{3}, 1 + \sqrt{3}), (2 - \sqrt{3}, 1 - \sqrt{3})$

9. $x^2 - 12x - 2 - \frac{10}{x - 1}$

10. $-1, 0.26794919, 3.7320508$

11. $(-1.292402, -2.292402), (0.39729507, -0.6027049), (3.8951065, 2.8951065)$

12. $\frac{acR_2}{kmR_2 + bkR_2 - bc}$

13. 2

14. $\frac{43\sqrt{21}}{21}$

15. $x^3y^{11/4}$

16. $\frac{5}{8}$

17. $\frac{m^2 - my}{mx - xy + mp}$

18. $(ab - 2x^2y^3)(a^2b^2 + 2abx^2y^3 + 4x^4y^6)$

19. $x(2x - 1)(x + 2)$

20. 10

21. 10,660

22. $a - b$

23. $\frac{n!}{n + 2}$

24. $\frac{8}{27}\pi\text{ cm}^3 \approx 0.9308\text{ cm}^3$

25. D

Problem Set 4

1. $-2.618034, -0.618034, -0.381966, 1.618034$

2. $(-2.377203, -3.377203), (-1.273891, -2.273891), (0, -1), (1.6510934, 0.65109341)$

3. $-\frac{1}{4}$

4. $\frac{10}{3}$

5. $-\frac{3\sqrt{3}}{2} + 1$

6. $4 - \frac{\sqrt{2}}{2}$

7. $\cos\theta$

8. $\sin^2\theta$

9. If a function is one-to-one, then it is not both increasing and decreasing.

10. Contrapositive: If your thumb does not hurt, then you did not hit your thumb with a hammer.

    Converse: If your thumb hurts, then you hit your thumb with a hammer.

    Inverse: If you did not hit your thumb with a hammer, then your thumb does not hurt.