Algebra $\frac{1}{2}$

An Incremental Development

THIRD EDITION

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LESSON 1 *Whole Number Place Value • Expanded Notation • Reading and Writing Whole Numbers • Addition*

1.A whole number place value

We use the **Hindu-Arabic system** to write our numbers. This system is a base 10 system and thus has ten different symbols. The symbols are called **digits,** or **numerals,** and they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

The numbers we say when we count are called **counting numbers,** or **natural numbers.** We may show the set of counting numbers this way:

{1, 2, 3, 4, 5, ...}

The three dots, called an *ellipsis*, mean that the list continues without end. The symbols, { }, are called braces and are sometimes used to designate a set. If we include zero with the set of counting numbers, then we form the set of **whole numbers.**

{0, 1, 2, 3, 4, ...}

When we write whole numbers, we can write the **decimal point** at the end of the number, or we can leave it off. Both of these

427. 427

represent the same number. In the right-hand number, the decimal point is assumed to be after the 7.

The value of a digit in a number depends on where the digit appears in the number. The first place to the left of the decimal point is the ones' place. We also call this place the **units' place,** which has a **place value** of 1. The next place to the left of the units' place is the **tens' place,** with a place value of 10, followed by the **hundreds' place,** with a place value of 100, and then the **thousands' place,** with a place value of 1,000. Each place to the left has one more zero.

Whole Number Place Values

100,000,000,000,000	10,000,000,000,000	1,000,000,000,000	,	100,000,000,000	10,000,000,000	1,000,000,000	,	100,000,000	10,000,000	1,000,000	,	100,000	10,000	1,000	,	100	10	1	.
hundred trillions	ten trillions	trillions		hundred billions	ten billions	billions		hundred millions	ten millions	millions		hundred thousands	ten thousands	thousands		hundreds	tens	units	decimal point

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To find the value of a digit in a number, multiply the digit times the place value. For example, the 5 in the left-hand number below

415,623 701,586 731,235

has a value of 5×1000, or 5000, because it is in the thousands' place. The value of the 5 in the center number is 5×100, or 500, because it is in the hundreds' place. The value of the 5 in the right-hand number is 5×1, or 5, because it is in the units' (ones') place.

example 1.1 In the number 46,235:

(a) What is the value of the digit 5?

(b) What is the value of the digit 2?

(c) What is the value of the digit 4?

solution First we write the decimal point at the end of the number.

46,235.

(a) The 5 is one place to the left of the decimal point. This is the units' place. This digit has a value of 5×1, or **5.**

(b) The 2 is three places to the left of the decimal point. This is the hundreds' place. This digit has a value of 2×100, or **200.**

(c) The 4 is five places to the left of the decimal point. This is the ten-thousands' place. This digit has a value of $4 \times 10{,}000$, or **40,000.**

1.B expanded notation

Writing a number in **expanded notation** is a good way to practice the idea of place value. When we write a number in expanded notation, we consider the value of every digit in the number individually. To write a number in expanded notation, we write each of the nonzero digits multiplied by the place value of the digit. We use parentheses to enclose each of these multiplications and put a plus sign between each set of parentheses.

To write 5020 in expanded notation, we write

$$(5 \times 1000) + (2 \times 10)$$

because this number contains five thousands and two tens.

example 1.2 Write the following number in standard notation: $(4 \times 10{,}000) + (6 \times 100) + (5 \times 1)$

solution Standard notation is our usual way of writing numbers. The number has four ten thousands, no thousands, six hundreds, no tens, and five ones. The number is **40,605.**

example 1.3 Write the number 6,305,126 in expanded notation.

solution

There are six millions, $(6 \times 1{,}000{,}000)$	three hundred thousands, $(3 \times 100{,}000)$		
five thousands, (5×1000)	one hundred, (1×100)	two tens, (2×10)	and six ones. (6×1)

If we add them all together, we get

$$\mathbf{(6 \times 1{,}000{,}000) + (3 \times 100{,}000) + (5 \times 1000) + (1 \times 100) + (2 \times 10) + (6 \times 1)}$$

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1.C reading and writing whole numbers

We begin by noting that all numbers between 20 and 100 that do not end in zero are hyphenated words when we write them out.

23 is written twenty-three	64 is written sixty-four
35 is written thirty-five	79 is written seventy-nine
42 is written forty-two	86 is written eighty-six
51 is written fifty-one	98 is written ninety-eight

The hyphen is also used in whole numbers when the whole number is used as a modifier. The words

ten thousand

are not hyphenated. But when we use these words as a modifier, as when we say

ten-thousands' place,

the words are hyphenated. Other examples of this rule are

hundred-millions' digit

ten-billions' place

hundred-thousands' place

The word *and* is not used when we write out whole numbers.

501	is written	five hundred one
	not	five hundred and one
370	is written	three hundred seventy
	not	three hundred and seventy
422	is written	four hundred twenty-two
	not	four hundred and twenty-two

Do not think of this as a useless exercise! Knowing how to correctly and accurately write numbers is necessary when writing a check, for example. Before we read whole numbers, we place a comma after every third digit beginning at the decimal point and moving to the left.† The commas divide the numbers into groups of three digits.

Place Value

Trillions			Billions			Millions			Thousands			Units (Ones)			Decimal point
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	·

To read the number 4125678942, we begin on the right-hand end, write a decimal point, and separate the number into groups of three by writing commas.

4,125,678,942.

Then we read the number, beginning with the leftmost group. First we read the number in the group, and then we read the name of the group. Then we move to the right and repeat the procedure.

four billion, one hundred twenty-five million, six hundred seventy-eight thousand, nine hundred forty-two

† It is our convention to usually write four-digit whole numbers without commas.

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example 1.4 Use words to write this number: 51723642

solution We write the decimal point on the right-hand end. Then we move to the left and place a comma after each group of three digits.

51,723,642.

The leftmost group is the millions' group. We read it as

fifty-one million,

and write the comma after the word *million*. The next three-digit group is the thousands' group. We read it as

seven hundred twenty-three thousand,

and we write the comma after the word *thousand*. The last three-digit group is the units' group. We do not say "units" but just read the three-digit number as

six hundred forty-two

Note that the words *fifty-one*, *twenty-three*, and *forty-two* are hyphenated. Also note that we do not use the word *and* between the groups. Now we put the parts together and read the number as

fifty-one million, seven hundred twenty-three thousand, six hundred forty-two

Note that the commas appear in the same places the commas appeared when we used digits to write the number.

example 1.5 Use digits to write the number fifty-one billion, twenty-seven thousand, five hundred twenty.

solution The first group is the billions' group. It contains the number fifty-one.

51, , ,

All the groups after the first group must have three digits. There are no millions, so we use three 0's.

51,000, ,

There are twenty-seven thousands. We write 027 in the next group so that the group will contain three digits.

51,000,027,

Now we finish by writing 520 in the last group.

51,000,027,520

1.D addition

When we add numbers, we call each of the numbers **addends,** and we call the answer a **sum.**

$$\begin{array}{rl} 523 & \text{addend} \\ 619 & \text{addend} \\ +\ 512 & \text{addend} \\ \hline 1654 & \text{sum} \end{array}$$

To add whole numbers, we write the numbers so that the units' places of the numbers are aligned vertically. Then we add in columns.

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example 1.6 Add: 4 + 407 + 3526

solution We write the numbers so that the units' places of the numbers are aligned vertically. Then we add.

$$\begin{array}{r} 4 \\ 407 \\ +\ 3526 \\ \hline 3937 \end{array}$$

To add money, we write each amount of money with a dollar sign, and with two places to the right of the decimal point. Then we align the decimal points and add.

example 1.7 Add: \$2.54 + \$5 + 9¢

solution We write the numbers so that there are two places to the right of each decimal point and align the decimal points. We include dollar signs, and then we add.

$$\begin{array}{r} \$2.54 \\ \$5.00 \\ +\ \$0.09 \\ \hline \$7.63 \end{array}$$

This book is designed to permit the reader to automate the upper-level skills of arithmetic while the concepts of algebra are being introduced. The addition, subtraction, multiplication, and division problems in the practice and problem sets are designed to provide paper-and-pencil practice in the four basic operations of arithmetic. **Throughout this book, do not use a calculator unless you are instructed to do so by your teacher.**

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practice

a. In the number 152068, what is the value of the 2?

b. Write in standard notation: $(6 \times 1000) + (4 \times 10) + (3 \times 1)$

c. Write 85,020 in expanded notation.

d. Use digits to write this number: ten billion, two hundred five million, forty-one thousand, five hundred

e. Use words to write this number: 36025103

problem set 1

1. In the number 5062973, what is the value of each of these digits?

(a) 6 (b) 9 (c) 3

2. Write the six-digit number that has the digit 4 in the thousands' place, with each of the remaining digits being 3.

3. Write the seven-digit number that has the digit 3 in the millions' place and the digit 7 in the hundreds' place, with each of the remaining digits being 6.

4. A number has eight digits. Every digit is 9 except the ten-millions' digit, which is 3, the ten-thousands' digit, which is 5, and the units' digit, which is 2. Use digits to write the number.

5. Use digits to write this number: forty-one billion, two hundred thousand, five hundred twenty

6. Use digits to write this number: five hundred seven billion, six hundred forty million, ninety thousand, forty-two

7. Use digits to write this number: four hundred seven trillion, ninety million, seven hundred forty-two thousand, seventy-two

8. Use digits to write this number: nine hundred eighty million, four hundred seventy

Use words to write each number:

9. 517236428

10. 90807060

11. 32000000652

12. 3250009111

13. 6040000

14. 99019900

Write each number in standard notation:

15. $(3 \times 100{,}000) + (4 \times 1000) + (2 \times 10)$

16. $(7 \times 10{,}000) + (8 \times 100) + (6 \times 10)$

17. $(9 \times 1000) + (4 \times 100) + (5 \times 1)$

18. $(7 \times 1{,}000{,}000) + (2 \times 10{,}000) + (6 \times 1000)$

Write each number in expanded notation:

19. 5280

20. 408

21. 70,600

22. 21,000

23. 4005

24. 9080

Add:

25.
$$\begin{array}{r} 43 \\ 76 \\ 84 \\ +\ 91 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 4628 \\ 5734 \\ +\ 8416 \\ \hline \end{array}$$

27.
$$\begin{array}{r} \$53.58 \\ +\ \$52.78 \\ \hline \end{array}$$

28. 9056 + 4708 + 9076

29. 432 + 846 + 943 + 721

30. \$3.64 + 52¢ + \$9

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