SAXON Math™

HOMESCHOOL

8/7

with Prealgebra

Hake
Saxon

CONTENTS

LETTER FROM AUTHOR STEPHEN HAKE

Dear Student,

We study mathematics because of its importance to our lives. Our study schedule, our trip to the store, the preparation of our meals, and many of the games we play involve mathematics. You will find that the word problems in this book are often drawn from everyday experiences.

As you grow into adulthood, mathematics will become even more important. In fact, your future in the adult world may depend on the mathematics you have learned. This book was written to help you learn mathematics and to learn it well. For this to happen, you must use the book properly. As you work through the pages, you will see that similar problems are presented over and over again. ***Solving each problem day after day is the secret to success.***

Your book is made up of daily lessons and investigations. Each lesson has four parts. The first part is a Warm-Up that includes practice of basic facts and mental math. These exercises improve your speed, accuracy, and ability to do math "in your head." The Warm-Up also includes a problem-solving exercise to familiarize you with strategies for solving complicated problems. The second part of the lesson is the New Concept. This section introduces a new mathematical concept and presents examples that use the concept. In the next section, the Lesson Practice, you have a chance to solve problems involving the new concept. The problems are lettered a, b, c, and so on. The final part of the lesson is the Mixed Practice. This problem set reviews previously taught concepts and prepares you for concepts that will be taught in later lessons. Solving these problems helps you remember skills and concepts for a long time.

Investigations are variations of the daily lesson that often involve activities. Investigations contain their own set of questions instead of a problem set.

Remember, solve every problem in every practice set, every problem set, and every investigation. Do not skip problems. With honest effort, you will experience success and true learning that will stay with you and serve you well in the future.

Stephen Hake
Temple City, California

LIST OF MATERIALS

The following materials are used throughout *Saxon Math 8/7—Homeschool*. We suggest you acquire these materials before beginning the program.

- inch/centimeter ruler

  (*Note:* a ruler that shows both customary and metric scales is preferred. However, separate customary and metric rulers are acceptable.)
- scientific calculator
- protractor
- graph paper (grid paper)
- compass (for drawing circles)
- scissors

Certain lessons and investigations contain activities that call for additional materials. Refer to the following list before beginning the specified lessons/investigations.

Investigation 1

- envelope or zip-top plastic bag (optional)
- colored pencils or markers (optional)

Lesson 61

- two pairs of plastic straws (The straws within a pair must be the same length. The two pairs may be different lengths.)
- thread or lightweight string
- paperclip (optional)

Lesson 66

- tape measure (preferably metric)
- circular objects

Lesson 70

- yardstick

Lesson 89

- length of string
- chalk
- masking tape (optional)

Lesson 97

- yardstick, ruler, and/or tape measure

Investigation 10

- pair of dot cubes

Investigation 11

- tape

Lesson 112

- two full length unsharpened pencils (or other straightedges)

Investigation 12

- envelope or zip-top plastic bag

LESSON

1 Arithmetic with Whole Numbers and Money • Variables and Evaluation

WARM-UP†

Facts Practice: 64 Multiplication Facts (Test A)

Mental Math: A score is 20. Two score and 4 is 44. How many is

a. 3 score **b.** 4 score **c.** 4 score and 7

d. Half a dozen **e.** 2 dozen **f.** 4 dozen

g. Start with a score. Add a dozen; divide by 4; add 2; then divide by 2. What is the answer?

Problem Solving:

What are the next three numbers in this pattern?

1, 3, 6, 10, 15, ...

NEW CONCEPTS

Arithmetic with whole numbers and money

The numbers we say when we count are called **counting numbers** or **natural numbers.** We can show the set of counting numbers this way:

{1, 2, 3, 4, 5, ...}

The three dots, called an *ellipsis,* mean that the list is infinite (goes on without end). The symbols { } are called *braces.* One use of braces is to designate a set. Including zero with the set of counting numbers forms the set of **whole numbers.**

{0, 1, 2, 3, 4, ...}

The set of whole numbers does not include any numbers less than zero, between 0 and 1, or between any **consecutive** counting numbers.

The four fundamental **operations of arithmetic** are addition, subtraction, multiplication, and division. In this lesson we will review the operations of arithmetic with whole numbers and with money. Amounts of money are sometimes indicated with a dollar sign ($) or with a cent sign (¢), but not both. We can show 50 cents either of these two ways:

$0.50 or 50¢

†For instructions on how to use the Warm-up activities, please consult the preface.

Occasionally we will see a dollar sign or cent sign used incorrectly.

This sign is incorrect because it uses a **decimal point** with a cent sign. This incorrect sign literally means that soft drinks cost not half a dollar but half a cent! Take care to express amounts of money in the proper form when performing arithmetic with money.

Numbers that are added are called **addends**, and the result of their addition is the **sum**.

$$\text{addend} + \text{addend} = \text{sum}$$

Example 1 Add:

(a) 36 + 472 + 3614

(b) \$1.45 + \$6 + 8¢

Solution (a) We align the digits in the ones place and add in columns. Looking for combinations of digits that total 10 may speed the work.

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 36 \\ 472 \\ +\ 3614 \\ \hline \mathbf{4122} \end{array}$$

(b) We write each amount of money with a dollar sign and two places to the right of the decimal point. We align the decimal points and add.

$$\begin{array}{r} {\scriptstyle 1} \\ \$1.45 \\ \$6.00 \\ +\ \$0.08 \\ \hline \mathbf{\$7.53} \end{array}$$

In subtraction the **subtrahend** is taken from the **minuend**. The result is the **difference**.

$$\text{minuend} - \text{subtrahend} = \text{difference}$$

Example 2 Subtract:

(a) 5207 − 948

(b) \$5 − 25¢

Solution (a) We align the digits in the ones place. We must follow the correct order of subtraction by writing the minuend (first number) above the subtrahend (second number).

$$\begin{array}{r} {\scriptstyle 4\,\overset{1}{1}\,9\,\overset{1}{1}} \\ 5\,\cancel{2}\,\cancel{0}\,7 \\ -\ \ 9\,4\,8 \\ \hline \mathbf{4\,2\,5\,9} \end{array}$$

(b) We write each amount in dollar form. We align decimal points and subtract.

$$\begin{array}{r} {\scriptstyle 4\ 9\ 1} \\ \$5.\cancel{0}0 \\ -\ \$0.25 \\ \hline \$4.75 \end{array}$$

Numbers that are multiplied are called **factors.** The result of their multiplication is the **product.**

$$\text{factor} \times \text{factor} = \text{product}$$

We can indicate the multiplication of two factors with a times sign, with a center dot, or by writing the factors next to each other with no sign between them.

$$4 \times 5 \qquad 4 \cdot 5 \qquad 4(5) \qquad ab$$

The parentheses in 4(5) clarify that 5 is a quantity separate from 4 and that the two digits do not represent the number 45. The expression ab means "a times b."

Example 3 Multiply:

(a) $164 \cdot 23$

(b) $\$4.68 \times 20$

(c) $5(29¢)$

Solution (a) We usually write the number with the most digits on top. We first multiply by the 3 of 23. Then we multiply by the 20 of 23. We add the partial products to find the final product.

$$\begin{array}{r} 164 \\ \times\ 23 \\ \hline 492 \\ 328 \\ \hline \mathbf{3772} \end{array}$$

(b) We can let the zero in 20 "hang out" to the right. We write 0 below the line and then multiply by the 2 of 20. We write the product with a dollar sign and two decimal places.

$$\begin{array}{r} \$4.68 \\ \times\ \ 20 \\ \hline \mathbf{\$93.60} \end{array}$$

(c) We can multiply 29¢ by 5 or write 29¢ as \$0.29 first. Since the product is greater than \$1, we use a dollar sign to write the answer.

$$\begin{array}{r} 29¢ \\ \times\ 5 \\ \hline 145¢ = \mathbf{\$1.45} \end{array}$$

In division the **dividend** is divided by the **divisor**. The result is the **quotient**. We can indicate division with a division sign (÷), a division box ( )⌐, or a division bar (–).

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

$$\text{divisor}\overline{)\text{dividend}}^{\text{quotient}} \qquad \frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

Example 4 Divide:

(a) $1234 \div 56$

(b) $\dfrac{\$12.60}{5}$

Solution (a) In this division there is a remainder. Other methods for dealing with a remainder will be considered later.

$$\begin{array}{r} \textbf{22 R 2} \\ 56\overline{)1234} \\ \underline{112} \\ 114 \\ \underline{112} \\ 2 \end{array}$$

(b) We write the quotient with a dollar sign. The decimal point in the quotient is directly above the decimal point in the dividend.

$$\begin{array}{r} \textbf{\$2.52} \\ 5\overline{)\$12.60} \\ \underline{10} \\ 26 \\ \underline{25} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Variables and evaluation

In mathematics, letters are often used to represent numbers—in formulas and expressions, for example. The letters are called **variables** because their values are not constant; rather, they vary. We **evaluate** an expression by calculating its value when the variables are assigned specific numbers.

Example 5 Evaluate each expression for $x = 10$ and $y = 5$:

(a) $x + y$ (b) $x - y$

(c) xy (d) $\dfrac{x}{y}$

Solution We substitute 10 for x and 5 for y in each expression. Then we perform the calculation.

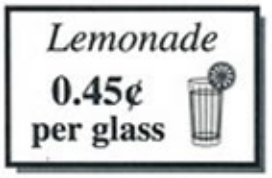
(a) $10 + 5 = \mathbf{15}$ (b) $10 - 5 = \mathbf{5}$

(c) $10 \cdot 5 = \mathbf{50}$ (d) $\dfrac{10}{5} = \mathbf{2}$

LESSON PRACTICE

Practice set

a. This sign is incorrect. Show two ways to correct the sign.



b. Name a whole number that is not a counting number.

c. When the product of 4 and 4 is divided by the sum of 4 and 4, what is the quotient?

Simplify by adding, subtracting, multiplying, or dividing as indicated:

d. $\$1.75 + 60¢ + \$3$

e. $\$2 - 47¢$

f. $5(65¢)$

g. $250 \cdot 24$

h. $\$24.00 \div 5$

i. $\frac{234}{18}$

Evaluate each expression for $a = 20$ and $b = 4$:

j. $a + b$

k. $a - b$

l. ab

m. $\frac{a}{b}$

MIXED PRACTICE

Problem set

1. When the sum of 5 and 6 is subtracted from the product of 5 and 6, what is the difference?

2. If the subtrahend is 9 and the difference is 8, what is the minuend?

3. If the divisor is 4 and the quotient is 8, what is the dividend?

4. When the product of 6 and 6 is divided by the sum of 6 and 6, what is the quotient?

5. Name the four fundamental operations of arithmetic.

6. Evaluate each expression for $n = 12$ and $m = 4$:

   (a) $n + m$

   (b) $n - m$

   (c) nm

   (d) $\frac{n}{m}$

Simplify by adding, subtracting, multiplying, or dividing, as indicated:

7. $\begin{array}{r} \$43.74 \\ -\ \$16.59 \\ \hline \end{array}$

8. $\begin{array}{r} 64 \\ \times\ 37 \\ \hline \end{array}$

9. $\begin{array}{r} 7 \\ 8 \\ 4 \\ 6 \\ 9 \\ 3 \\ 5 \\ +\ 7 \\ \hline \end{array}$

10. 364 + 52 + 867 + 9

11. 4000 − 3625

12. (316)(18)

13. \$43.60 ÷ 20

14. 300 · 40

15. 8 · 12 · 0

16. 3708 ÷ 12

17. 365 × 20

18. $25\overline{)767}$

19. 30(40)

20. \$10 − \$2.34

21. 4017 − 3952

22. \$2.50 × 80

23. 20(\$2.50)

24. $\frac{560}{14}$

25. $\frac{\$10.00}{8}$

26. What is another name for *counting numbers?*

27. Write 25 cents twice, once with a dollar sign and once with a cent sign.

28. Which counting numbers are also whole numbers?

29. What is the name for the answer to a division problem?

30. Here we use a plus sign and an equal sign to show the relationship of addends and their sum:

addend + addend = sum

Use a minus sign, an equal sign, and the words *difference, subtrahend,* and *minuend* to show the relationship between the numbers in subtraction.

LESSON

55 Average, Part 2

WARM-UP

Facts Practice: + − × ÷ Decimals (Test J)

Mental Math:

a. $20 \times \$0.25$

b. 0.375×10^2

c. $2x - 5 = 75$

d. Convert 3000 m to km.

e. $\left(\frac{2}{3}\right)^2$

f. $\frac{3}{4}$ of 100

g. At 30 pages an hour, how many pages can Mike read in $2\frac{1}{2}$ hours?

Problem Solving:

Copy this problem and fill in the missing digits:

$$\begin{array}{r} \_3\_ \\ \times\ \_\_ \\ \hline 3\_\_\_ \\ \_3\_ \\ \hline 9\_\_9 \end{array}$$

NEW CONCEPT

If we know the average of a group of numbers and how many numbers are in the group, we can determine the sum of the numbers.

Example 1 The average of three numbers is 17. What is their sum?

Solution We are not told what the numbers are, only their average. Each of these sets of three numbers has an average of 17:

$$\frac{16 + 17 + 18}{3} = \frac{51}{3} = 17$$

$$\frac{10 + 11 + 30}{3} = \frac{51}{3} = 17$$

$$\frac{1 + 1 + 49}{3} = \frac{51}{3} = 17$$

LESSON

120 Graphing Nonlinear Equations

WARM-UP

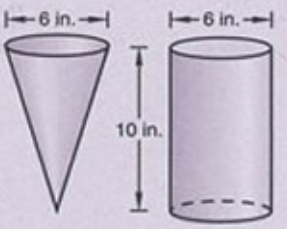
Facts Practice: Multiplying and Dividing in Scientific Notation (Test W)

Mental Math:

a. 1000001 (base 2)
b. MCMLXIX
c. $(10^2)(10^{-2})$
d. $(5 \times 10^{-5})^2$
e. $2x^2 = 32$
f. Convert 0°C to Fahrenheit.
g. 10% of $250
h. 10% more than $250
i. 2 × 12, + 1, √, × 3, + 1, √, × 2, + 1, √, + 1, √, − 1, √

Problem Solving:

A paper cone is filled with water. Then the water is poured into a cylindrical glass beaker that has the same height and diameter as the paper cone. How many cones of water are needed to fill the beaker?



NEW CONCEPT

Equations whose graphs are lines are called **linear equations.** (Notice the word *line* in *linear*.) In this lesson we will graph equations whose graphs are not lines but are curves. These equations are called **nonlinear equations.** To graph each nonlinear equation, we will make a table of ordered pairs and plot enough points to get an idea of the path of the curve.

Example 1 Graph: $y = \frac{6}{x}$

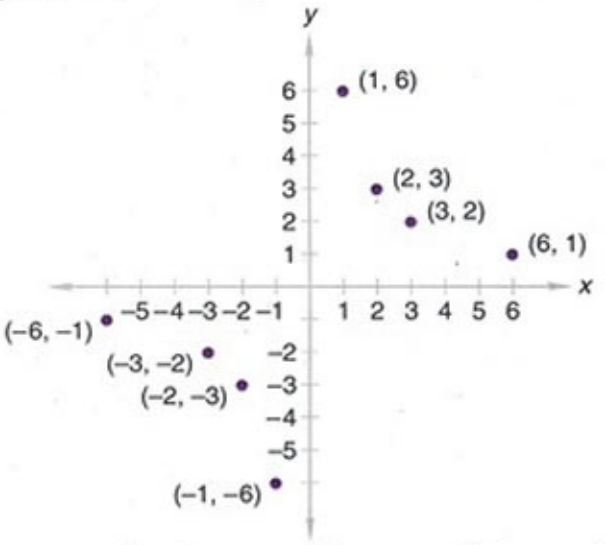
Solution We make a table of ordered pairs. For convenience we select x values that are factors of 6. We remember to select negative values as well. Note that we may not select zero for x.

$y = \frac{6}{x}$

x	y	(x, y)
1	6	(1, 6)
2	3	(2, 3)
3	2	(3, 2)
6	1	(6, 1)

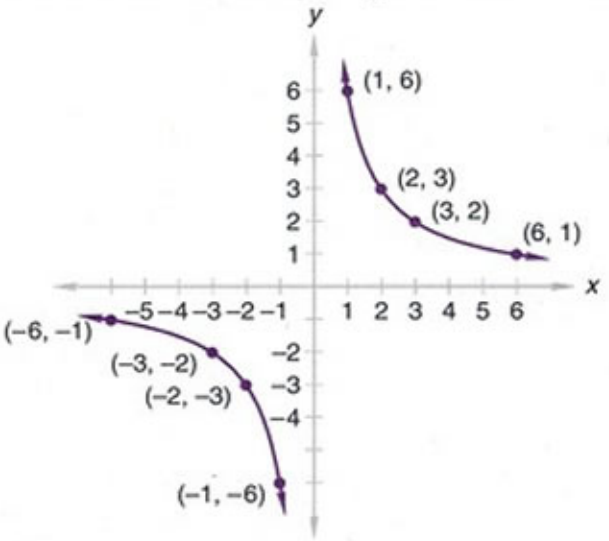
x	y	(x, y)
−1	−6	(−1, −6)
−2	−3	(−2, −3)
−3	−2	(−3, −2)
−6	−1	(−6, −1)

On a coordinate plane we graph the x, y pairs we found that satisfy the equation.



This arrangement of points on the coordinate plane suggests two curves that do not intersect.

We draw two smooth curves through the two sets of points.



Example 2 Graph: $y = x^2$

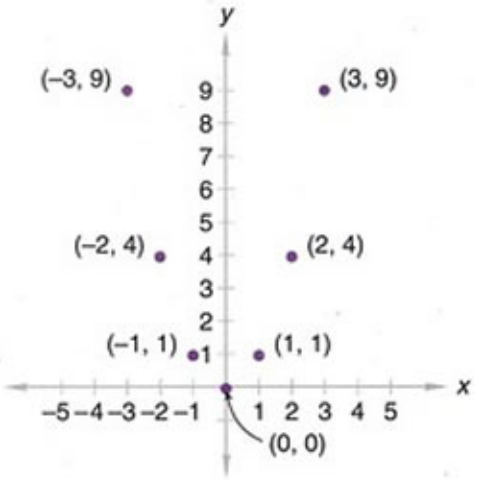
Solution We begin by making a table of ordered pairs. We think of numbers for x and then calculate y. We replace x with negative numbers as well. Remember that squaring a negative number results in a positive number.

$$y = x^2$$

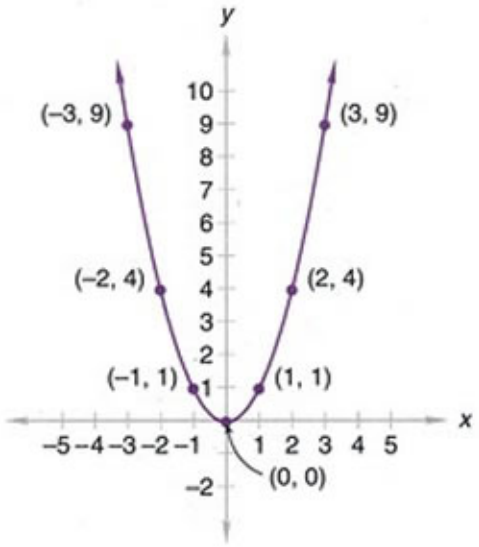
x	y	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

x	y	(x, y)
−1	1	(−1, 1)
−2	4	(−2, 4)
−3	9	(−3, 9)

After generating several pairs of coordinates, we graph the points on a coordinate plane.



We complete the graph by drawing a smooth curve through the graped points.



The coordinates of any point on the curve should satisfy the original equation.

LESSON PRACTICE

Practice set

a. Graph $y = \frac{12}{x}$. Begin by creating a table of ordered pairs. Use 6, 4, 3, 2, –2, –3, –4, and –6 in place of x.

b. Graph $y = x^2 - 2$. Compare your graph to the graph in example 2.

c. Graph $y = \frac{10}{x}$. Compare your graph to the graph in example 1.

d. Graph $y = 2x^2$. Compare your graph to the graph in example 2.

MIXED PRACTICE

Problem set

1. *(Inv. 10)* Schuster was playing a board game and rolled a 7 with a pair of dot cubes three times in a row. What are the odds of Schuster rolling a 7 with the next roll of the dot cubes?

2. *(92)* If the total cost of an item including 8% sales tax is \$2.70, then what was the price before tax was added?

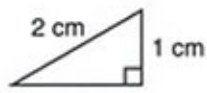
3. *(79)* Compare: $x^2 \bigcirc y^2$ if $x < y$

4. *(58)* If a trapezoid has a line of symmetry and one of its angles measures 100°, what is the measure of each of its other angles?

5. *(48)* Complete the table.

FRACTION	DECIMAL	PERCENT
(a)	(b)	0.1%
$\frac{8}{5}$	(c)	(d)

6. *(99)* The hypotenuse of this triangle is twice the length of the shorter leg.



(a) Use the Pythagorean theorem to find the length of the remaining side.

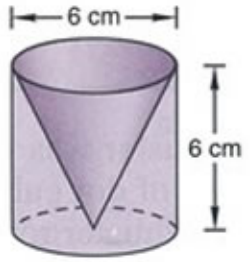
(b) Use a centimeter ruler to find the length of the unmarked side to the nearest tenth of a centimeter.

7. *(111)* Simplify. Write the answer in scientific notation.

$$\frac{(4 \times 10^{-5})(6 \times 10^{-4})}{8 \times 10^{3}}$$

8. Factor each expression:
(116)
(a) $3y^2 - y$ (b) $6w^2 + 9wx - 12w$

The figure below shows a cylinder and a cone whose heights and diameters are equal. Refer to the figure to answer problems 9 and 10.



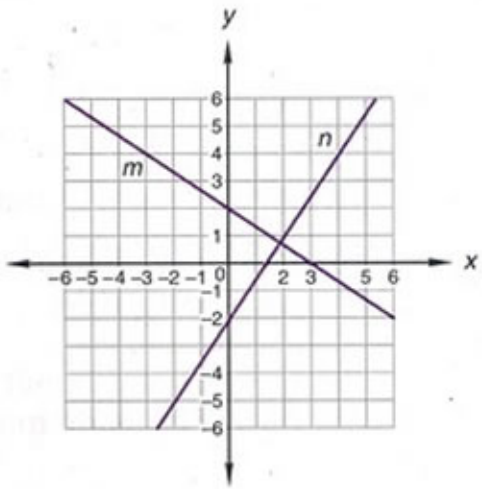
9. What is the ratio of the volume of the cone to the volume of the cylinder?
(113)

10. The lateral surface area of a cylinder is the area of the curved side and excludes the areas of the circular ends. What is the lateral surface area of the cylinder rounded to the nearest square centimeter? (Use 3.14 for π.)
(105)

11. Transform the formula $E = mc^2$ to solve for m.
(106)

12. If 60% of the children at the theater were girls, what was the ratio of boys to girls at the theater?
(54)

The graph below shows $m \perp n$. Refer to the graph to answer problems 13 and 14.

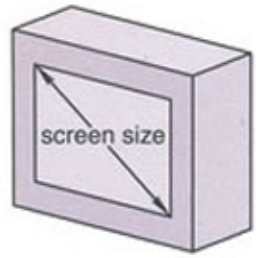


13. What is the equation of each line in slope-intercept form?
(117)

14. What is the product of the slopes of lines m and n? Why?
(107)

15. (110) If a $1000 investment earns 20% interest compounded annually, then the investment will double in value in how many years?

16. (112) The stated size of a TV screen or computer monitor is its diagonal measure. A screen that is 17 in. wide and 12 in. tall would be described as what size of screen? Round the answer to the nearest inch.



17. (70, 88) Premixed concrete is sold by the cubic yard. The Smiths are pouring a concrete driveway that is 36 feet long, 21 feet wide, and $\frac{1}{2}$ foot thick.

(a) Find the number of cubic feet of concrete needed.

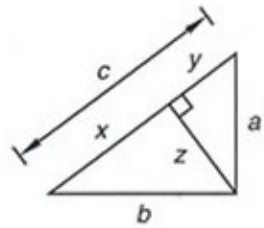
(b) Use three unit multipliers to convert answer (a) to cubic yards.

18. (119) In the following expressions, what number may not be used for the variable?

(a) $\dfrac{12}{4 - 2m}$ (b) $\dfrac{y - 5}{y + 5}$

19. (120) Graph: $y = x^2 - 4$

20. (97) Refer to this drawing of three similar triangles to find the letter that completes the proportion below.



$$\frac{c}{a} = \frac{a}{?}$$

21. (105) Recall that the surface area of a sphere is four times the area of its largest "cross section." What is the approximate surface area of a cantaloupe that is 6 inches in diameter? Use 3.14 for π and round the answer to the nearest square inch.

22. (115) A cup containing 250 cubic centimeters of water holds how many liters of water?