



Algebra 1

An Incremental Development
THIRD EDITION

Homeschool Packet

Algebra 1

An Incremental Development

Test Forms

Algebra 1

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SAXON

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LESSON 1 Addition and Subtraction of Fractions • Lines and Segments

1.A

addition and subtraction of fractions

To add or subtract fractions that have the same denominators, we add or subtract the numerators as indicated below, and the result is recorded over the same denominator.

$$\frac{5}{11} + \frac{2}{11} = \frac{7}{11} \qquad \frac{5}{11} - \frac{2}{11} = \frac{3}{11}$$

If the denominators are not the same, it is necessary to rewrite the fractions so that they have the same denominators.

	PROBLEM	REWRITTEN WITH EQUAL DENOMINATORS	ANSWER
(a)	$\frac{1}{3} + \frac{2}{5}$	$\frac{5}{15} + \frac{6}{15}$	$\frac{11}{15}$
(b)	$\frac{2}{3} - \frac{1}{8}$	$\frac{16}{24} - \frac{3}{24}$	$\frac{13}{24}$

A **mixed number** is the sum of a whole number and a fraction. Thus the notation

$$13\frac{3}{5}$$

does not mean 13 multiplied by $\frac{3}{5}$ but instead 13 plus $\frac{3}{5}$.

$$13 + \frac{3}{5}$$

When we add and subtract mixed numbers, we handle the fractions and the whole numbers separately. In some subtraction problems it is necessary to borrow, as shown in (e).

	PROBLEM	REWRITTEN WITH EQUAL DENOMINATORS	ANSWER
(c)	$13\frac{3}{5} + 2\frac{1}{8}$	$13\frac{24}{40} + 2\frac{5}{40}$	$15\frac{29}{40}$
(d)	$13\frac{3}{5} - 2\frac{1}{8}$	$13\frac{24}{40} - 2\frac{5}{40}$	$11\frac{19}{40}$
		BORROWING	
(e)	$13\frac{3}{5} - 2\frac{7}{8}$	$13\frac{24}{40} - 2\frac{35}{40} = 12\frac{64}{40} - 2\frac{35}{40}$	$10\frac{29}{40}$

1.B

lines and segments

It is impossible to draw a mathematical line because a mathematical line is a **straight line** that has **no width** and **no ends**. To show the location of a mathematical line, we draw a pencil line and put arrowheads on both ends to emphasize that the mathematical line goes on and on in both directions.



We can name a line by naming any two points on the line and using an overbar with two arrowheads. We can designate the line shown by writing \overleftrightarrow{AX} , \overleftrightarrow{XA} , \overleftrightarrow{AC} , \overleftrightarrow{CA} , \overleftrightarrow{XC} , or \overleftrightarrow{CX} .

A part of a line is called a **line segment**. A line segment contains the endpoints and all points between the endpoints. To show the location of a line segment, we use a pencil line with no arrowheads. We name a segment by naming the endpoints of the segment.



This is segment MC or segment CM . We can indicate that two letters name a segment by using an overbar with no arrowheads. Thus \overline{MC} means segment MC . If we use two letters without the overbar, we designate the length of the segment. Thus MC is the length of \overline{MC} .

example 1.1 Add: $\frac{10}{11} - \frac{5}{6} + \frac{1}{3}$

solution We begin by rewriting each fraction so that they have the same denominators. Then we add the fractions.

$$\begin{aligned} \frac{10}{11} - \frac{5}{6} + \frac{1}{3} &= \frac{60}{66} - \frac{55}{66} + \frac{22}{66} && \text{common denominators} \\ &= \frac{5}{66} + \frac{22}{66} && \text{added} \\ &= \frac{27}{66} && \text{added} \\ &= \frac{9}{22} && \text{simplified} \end{aligned}$$

example 1.2 Segment AC measures $10\frac{1}{4}$ units. Segment AB measures $4\frac{3}{7}$ units. Find BC .



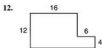
solution We need to know the length of segment BC . We know AC and AB . We subtract to find BC .

$$\begin{aligned} BC &= AC - AB \\ &= 10\frac{1}{4} - 4\frac{3}{7} && \text{substituted} \\ &= 10\frac{7}{28} - 4\frac{12}{28} && \text{common denominators} \\ &= 9\frac{35}{28} - 4\frac{12}{28} && \text{borrowed} \\ &= 5\frac{23}{28} \text{ units} && \text{subtracted} \end{aligned}$$

- What is the degree measure of a right angle?
 - What is the degree measure of a straight angle?
- Define *obtuse triangle*.
 - Define *scalene triangle*.
- What is the name of the parallelogram that has four right angles?
- What name is given to polygons whose sides all have the same length and whose angles all have the same measure?
- What is the sum of the measures of the three angles of any triangle?
- The radius of a circle is 5 centimeters. Find the circumference of the circle.
- The diameter of a circle is 12 inches. Find the circumference of the circle.
- Use two unit multipliers to convert 36 inches to yards. (Go from inches to feet to yards.)
- What is another name for the set of counting numbers?
- The perimeter of a square is 24 meters. What is the length of one side of the square?
- Find x .



Find the perimeters of the following figures. Corners that look square are square. Dimensions are in meters.



Add, subtract, multiply, or divide as indicated. Write the answers as proper fractions reduced to lowest terms or as mixed numbers.

14. $\frac{5}{2} \times \frac{4}{3} \times \frac{9}{10}$

15. $3\frac{2}{5} + 5\frac{3}{10}$

16. $3\frac{1}{2} + 5\frac{3}{5}$

17. $\frac{12\frac{1}{2}}{3\frac{1}{4}}$

18. $11,922 \div 2,3844$

19. $14\frac{3}{8} - 8\frac{9}{16}$

20. The length of \overline{AC} is $3\frac{3}{4}$ units. The length of \overline{AB} is $1\frac{1}{2}$ units. Find BC .



Test 1

1. (a) 90°

(b) 180°

2. (a) **Obtuse triangle: a triangle that has one angle with a measure greater than 90°** (b) **Scalene triangle: a triangle that has no sides of equal length**3. **Rectangle**4. **Regular polygons**5. 180°

6. $C = 2\pi r = 2\pi(5) = 10\pi \text{ cm}$

7. $C = \pi D = \pi(12) = 12\pi \text{ in.}$

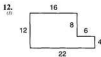
8. $36 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{36}{(12)(3)} \text{ yd}$

9. **Natural numbers**

10. $s = P + 4 = (24) + 4 = 6 \text{ m}$

11. $40 + 90 = 130$

$x = 180 - (130) = 50$



$P = 12 + 16 + 8 + 6 + 4 + 22 = 68 \text{ m}$



$$\begin{aligned}
 P &= 6 + 8 + 6 + \frac{2\pi(4)}{2} \\
 &= 20 + \frac{8\pi}{2} \\
 &= (20 + 4\pi) \text{ m}
 \end{aligned}$$

14. $\frac{5}{2} \times \frac{4}{3} \times \frac{9}{10} = 3$

15. $3\frac{2}{5} + 5\frac{3}{10} = 3\frac{4}{10} + 5\frac{3}{10} = 8\frac{7}{10}$

16. $3\frac{1}{2} + 5\frac{3}{5} = \frac{7}{2} + \frac{28}{5} = \frac{7}{2} \cdot \frac{5}{28} = \frac{5}{8}$

17. $\frac{12\frac{1}{2}}{3\frac{1}{4}} = \frac{\frac{25}{2}}{\frac{13}{4}} = \frac{25}{2} \cdot \frac{4}{13} = \frac{50}{13} = 3\frac{11}{13}$

18. $2.3844 \overline{) 11.922}$

$$\begin{array}{r}
 2.3844 \overline{) 11.9220} \\
 \underline{11.9220} \\
 0
 \end{array}$$

19. $14\frac{3}{8} - 8\frac{9}{16} = 14\frac{6}{16} - 8\frac{9}{16} = 13\frac{22}{16} - 8\frac{9}{16} = 5\frac{13}{16}$

20. $BC = 3\frac{2}{3} - 1\frac{5}{12} = 3\frac{8}{12} - 1\frac{5}{12} = 2\frac{3}{12}$
 $= 2\frac{1}{4} \text{ units}$

Test 2

1. (a) $\{1, 2, 3, 4, 5, \dots\}$ (b) $\{0, 1, 2, 3, 4, 5, \dots\}$ (c) $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ 2. (a) **Difference**(b) **Product**3. **Zero**

4. $35 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{35}{(2.54)(12)} \text{ ft}$

5. $250 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 250(3)(12) \text{ in.}$

6. $P = 2l + 2w = 2(10) + 2(5) = 20 + 10 = 30 \text{ m}$

7. (a) $C = 2\pi r = 2\pi(7) = 14\pi \text{ in.}$

(b) $A = \pi r^2 = \pi(7)^2 = 49\pi \text{ in.}^2$

8. $-(-4) + (-3) - (-2) = (+4) + (-3) + (+2) = 3$

9. $7 - 6 - (-3) - [-(-3)]$

$= (+7) + (-6) + (+3) + (-3) = 1$